Conservation laws and angular transverse shifts of the reflected and transmitted light beams

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Abstract

The relation between the angular transverse shifts of the beams reflected and transmitted at a plane interface of two isotropic transparent media is established. The evaluation of this relation is based on the conservation law of the transverse component of the Minkowski linear momentum, which takes place in the processes of the reflection and transmission of wave packets.

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I. Introduction

It is known that in some cases light beams undergo transverse shifts (TSs) after reflection and transmission at a plane interface of isotropic transparent media. Two types of TSs are known: the linear TS (LTS) and the angular TS (ATS).

The LTS phenomenon means that the axis of the secondary beam is parallel to the plane of incidence, but it does not lie in it. This effect is connected with the transformation of the spin or intrinsic orbital angular momentum at reflection and transmission of the wave packet. The spin-dependent LTS has been studied for more than half a century (see, for instance, [1-13] and the references therein). In the case of total reflection, this effect is usually called the Imbert-Fedorov shift. The LTSs of the reflected and transmitted beam, which depend on the intrinsic orbital angular momentum of the incident beam,

have been predicted in [14, 15]; the experimental investigations of this and the associated effects have been carried out in [16-18].

The ATS phenomenon means that the actual axis of the secondary beam is inclined to the plane of incidence, the value of the ATS is given by the angle of inclination. Like in the case of LTSs, two types of ATSs are known: the spin-dependent and the spin-independent ATSs. The expression for the spin-dependent ATS of the reflected beam has been obtained in the work by Nasalski [9]. Bliokh and Bliokh [13] have performed explicit calculations of spin-dependent ATSs for both reflected and transmitted beams. It should also be mentioned that the possibility of these ATSs has been pointed out in [7]. Spin-independent ATSs of the reflected and transmitted beams have been predicted in Ref. [14] and in the papers by Alda [19] and by Alda and Rico-Garcia [20].

The LTSs of the secondary beams are controlled by the conservation law of the normal-to-interface component of the Minkowski angular momentum [21-24], what is explained as follows. If the normal component of the total intrinsic angular momentum changes after reflection and transmission, the above-mentioned law demands that the centres of gravity of the secondary packets must undergo shifts in the transverse direction in order that the normal component of the total extrinsic AM generated by the shifts could compensate this change. In application to the totally reflected beam, this fact was known long ago [3]. As for the LTSs of partially reflected and transmitted beams, these phenomena have been especially intensively discussed from the point of view of the conservation laws in the last years [12, 13, 24].

In this communication, it will be shown that the ATSs of the secondary beams are also controlled by a conservation law, namely by conservation of the transverse, i.e. perpendicular to the plane of incidence, component of the Minkowski linear momentum.

II. Geometry of reflection and transmission

Let us consider the reflection and transmission of a monochromatic light beam at a plane interface of two semi-infinite transparent, isotropic, nondispersive, and nonmagnetic media. The scheme of this process is shown in Fig. 1. The position of the interface is defined by the equation $\hat{\mathbf{N}} \cdot \mathbf{x} = 0$, where $\mathbf{x}$ is the 3D radius vector, and $\hat{\mathbf{N}}$ is the unit normal to the interface directed from the first medium, which fills the upper half-space, to the second one. The beam is assumed to be incident from the first medium. Let us denote the refractive indices of the media in the upper half-space ($\hat{\mathbf{N}} \cdot \mathbf{x} < 0$) and the lower half-space ($\hat{\mathbf{N}} \cdot \mathbf{x} > 0$) by $n^{(1)}$ and $n^{(2)}$, respectively.

Throughout this paper, we shall use the superscripts i, ρ, and τ for indicating the quantities characteristic of the incident, reflected and transmitted beams, respectively. The superscript a will be used in order to designate an arbitrary

(incident, reflected or transmitted) beam and the superscript s in order to designate one of the secondary beams (reflected or transmitted). Thus, a=i, ρ, or τ, and s=ρ or τ. By $n^{(a)}$ the refractive index of the medium will be denoted, in which the a-th beam propagates, i.e. $n^{(i)} = n^{(\rho)} = n^{(1)}$, and $n^{(\tau)} = n^{(2)}$.

We shall employ four coordinate systems. Three systems are connected with the incident, reflected and transmitted beams. These systems will be termed the i-, ρ, and τ-systems, their bases are given by the unit vectors $\hat{\mathbf{x}}_1^{(a)}$, $\hat{\mathbf{x}}_2^{(a)}$, and $\hat{\mathbf{x}}_3^{(a)}$. In the i-system, the $x_3^{(i)}$ axis is assumed to coincide with the incident beam's axis (the exact definition of the latter will be given later). This axis and the unit vector $\hat{\mathbf{N}}$ define the position of the beam's plane of incidence in 3D space; the angle between the unit vectors $\hat{\mathbf{N}}$ and $\hat{\mathbf{x}}_3^{(i)}$ is the beam's angle of incidence: $\theta^{(i)} = \arccos(\hat{\mathbf{N}} \cdot \hat{\mathbf{x}}_3^{(i)})$. The coordinate origin O in every system is taken to be the point of intersection of the incident beam axis with the interface. In the ρ- and τ-systems, the $x_3^{(\rho)}$ and $x_3^{(\tau)}$ axes are assumed to coincide with the geometric-optical axes of the reflected and transmitted beams. The geometric-optical axes of the secondary beams are defined as the rays which are intersected by the interface at the coordinate origin and whose directions are characterized by the unit vectors $\hat{\mathbf{x}}_3^{(\rho)}$ and $\hat{\mathbf{x}}_3^{(\tau)}$, these vectors being related to $\hat{\mathbf{x}}_3^{(i)}$ through the Snell laws. The beam's angles of reflection and transmission $\theta^{(\rho)}$ and $\theta^{(\tau)}$ are the angles between the vector $\hat{\mathbf{N}}$ and the respective geometric optics axis. $\theta^{(\rho)} = \pi - \theta^{(i)}$, while $\theta^{(\tau)}$ is defined by the relation $n^{(2)} \sin\theta^{(\tau)} = n^{(1)} \sin\theta^{(i)}$.

Let us define the x_2 axis of every system to be perpendicular to the plane of incidence, this axis (the transverse one) is characterized by the unit vector $\hat{\mathbf{x}}_2 = \hat{\mathbf{N}}\times\hat{\mathbf{x}}_3^{(i)}/|\hat{\mathbf{N}}\times\hat{\mathbf{x}}_3^{(i)}|$. The $x_1^{(i)}$, $x_1^{(\rho)}$, and $x_1^{(\tau)}$ axes lie in the plane of incidence, the direction of the $x_1^{(a)}$ axis is given by the unit vector $\hat{\mathbf{x}}_1^{(a)} = \hat{\mathbf{x}}_2 \times \hat{\mathbf{x}}_3^{(a)}$.

In the a-th system, the 3D radius-vector is represented as follows:

$$\mathbf{x} = \mathbf{x}_\perp^{(a)} + x_3^{(a)}\hat{\mathbf{x}}_3^{(a)} \ , \tag{1}$$

where $\mathbf{x}_\perp^{(i)}$, $\mathbf{x}_\perp^{(\rho)}$, and $\mathbf{x}_\perp^{(\tau)}$ are the 2D planar radius vectors lying in the planes which are perpendicular to the axis of the incident beam or to the geometric optics axes of the respective secondary beam:

$$\mathbf{x}_\perp^{(a)} = x_1^{(a)}\hat{\mathbf{x}}_1^{(a)} + x_2\hat{\mathbf{x}}_2 \ . \tag{2}$$

The fourth system is connected with the interface, its basis being given by the unit vectors $\hat{\mathbf{x}}_1 = -\hat{\mathbf{N}}$, $\hat{\mathbf{x}}_2$, and $\hat{\mathbf{x}}_3 = \hat{\mathbf{x}}_1 \times \hat{\mathbf{x}}_2$.

III. Relation between the ATSs of the reflected and transmitted beams

We shall assume that the incident beam is paraxial, i.e. that $\lambda \ll b$, where

λ is the wavelength of the light in vacuum and b is a mean dimension of the beam in the planar plane.

Let us denote the electric and magnetic field vectors of the a-th beam at the point $\mathbf{x}$ by $\mathbf{E}^{(a)}(\mathbf{x})$ and $\mathbf{H}^{(a)}(\mathbf{x})$. The vector $\mathbf{F}^{(a)}(\mathbf{x})$ will designate either $\mathbf{E}^{(a)}(\mathbf{x})$ or $\mathbf{H}^{(a)}(\mathbf{x})$. The time dependence of the field vectors and of the other characteristics of the beams is suppressed.

In order to consider the ATS phenomenon, it is necessary to define the actual axes of the secondary beams. Let us first define the centre of gravity of the a-th beam. Its position on the beam's cross-section is given by the 2D vector

$$\mathbf{R}_{\perp}^{(a)}(X_3^{(a)}) = \frac{\int \mathbf{x}_{\perp}^{(a)} w^{(a)}(\mathbf{x}_{\perp}^{(a)}, X_3^{(a)}) d\mathbf{x}_{\perp}^{(a)}}{\int w^{(a)}(\mathbf{x}_{\perp}^{(a)}, X_3^{(a)}) d\mathbf{x}_{\perp}^{(a)}} \ , \tag{3}$$

where $X_3^{(a)}$ is the $x_3^{(a)}$-coordinate of the cross-section, and $w^{(a)}(\mathbf{x})$ is the electromagnetic energy density inside the a-th beam,

$$w^{(a)}(\mathbf{x}) = \frac{1}{8\pi}\left[(n^{(a)})^2\left(\mathbf{E}^{(a)}(\mathbf{x})\right)^2 + \left(\mathbf{H}^{(a)}(\mathbf{x})\right)^2\right] \ . \tag{4}$$

ATSs are actual in the far-field region. In view of that, let us assume that every $X_3^{(a)}$ satisfies the condition:

$$|X_3^{(a)}| > b^2/\lambda \ . \tag{5}$$

Let us also assume that, if $\theta^{(a)}$ is close to 0 or to $\pi/2$, $X_3^{(a)}$ satisfy the additional condition:

$$|X_3^{(a)}| \gg b/\sin(2\theta^{(a)}) \ . \tag{6}$$

If the relation (6) is fulfilled, the electromagnetic fields of the beams are negligible at the lines of intersection of the respective cross-sections and the interface. Again, the reflected field can be ignored on the cross-section of the incident beam and vice versa.

In the regions (5), the geometrical loci of the centres of gravity of the incident, reflected and transmitted beams are the straight lines. Let us define the first one to be the axis of the incident beam. The second and the third lines determine the actual axes of the secondary beams. These axes and the unit normal $\hat{\mathbf{N}}$ give the actual planes of reflection and transmission, the angles between them and the incidence plane will be denoted by $\Phi^{(\rho)}$ and $\Phi^{(\tau)}$, respectively. Let us define the ATSs of the reflected and transmitted beams, $\phi^{(\rho)}$ and $\phi^{(\tau)}$, as the angles of the inclination of the beams' actual axes to the incidence plane. The angles $\phi^{(s)}$ and $\Phi^{(s)}$ are small [13, 14, 19, 20], thus, $\sin\phi^{(s)} \simeq \phi^{(s)}$ and $\sin\Phi^{(s)} \simeq \Phi^{(s)}$. In this case, the relation between $\phi^{(s)}$ and $\Phi^{(s)}$ is as follows:

$$\Phi^{(s)} \simeq \frac{\phi^{(s)}}{\sin\theta^{(s)}} \ . \tag{7}$$

The meanings of the angles $\phi^{(\rho)}$ and $\Phi^{(\rho)}$ are illustrated in Fig. 2, the meanings of the angles $\phi^{(\tau)}$ and $\Phi^{(\tau)}$ are similar.

Let us now consider the ATS phenomenon from the point of view of the dynamics of the processes of the reflection and transmission of wave packets. In order to do that, let us fix a time instant $t_0 = n^{(1)} X_3^{(i)}$ $(t_0 < 0)$ and select at this instant a section of the incident beam, which is restricted with two cross-sections, the axial coordinate of its centre being equal to $X_3^{(i)}$ (see Fig. 3). Its length, i.e. the distance between the cross-sections, will be denoted by D. Let us impose the following restrictions on D:

$$|X_3^{(i)}| \gg D \gg \sqrt{\lambda |X_3^{(i)}|}\ . \tag{8}$$

Being separated from the beam, the selected section represents a packet of electromagnetic waves. Qualitatively, the process of propagation of the packet constructed in such a way is Fig. 3. At the instant t_0 and during some interval after it, the influence of the second medium on the incident field is negligible. Thus, during this interval, the motion of the packet is quasi-free, i.e. it occurs as if the packet propagates in the homogeneous medium with the refracted index $n^{(1)}$. At $|t| \sim n^{(1)} D$, the incident, reflected and transmitted fields coexist. Finally, when t is positive and large enough, the reflected and transmitted packets are well formed, and they propagate quasi-freely in the first and the second medium, respectively. Further on, the time instants, when the motions of the packets are quasi-free, will be specified by means of the accent "tilde" over t, these times obey the condition:

$$|\tilde{t}| \gg n^{(1)} D\ . \tag{9}$$

Let us now perform a detailed analysis of the motion of the constructed packet. Further on, the same letters will be used for notations of field vectors and other characteristics of the a-th beam and of the a-th packet. However, in order to make the difference between the characteristics of the beam and of the packet, in the latter case the respective letter will be overlined. Again, when necessary, the dependence of the respective characteristics on t will be explicitly pointed out.

Our analysis of the motion of the constructed packet will be based on the local energy-momentum conservation laws, which take place at an arbitrary $t > t_0$. These laws can be written by means of the Minkowski energy-momentum tensor $\overline{T}_{jk}(\mathbf{x}; t)$ [22, 25, 26]. In order to do this, let us introduce the conception of a total electromagnetic field, whose characteristics will be written down without superscript a. It is evident that

$$\overline{\mathbf{F}}(\mathbf{x}; t) = \overline{\mathbf{F}}^{(i)}(\mathbf{x}; t) + \overline{\mathbf{F}}^{(\rho)}(\mathbf{x}; t), \ \text{ if } x_1 > 0\ , \tag{10}$$

and

$$\overline{\mathbf{F}}(\mathbf{x}; t) = \overline{\mathbf{F}}^{(\tau)}(\mathbf{x}; t), \ \text{ if } x_1 < 0\ . \tag{11}$$

Again, let us denote by $\varepsilon(x_1)$ the dielectric constant at an arbitrary point in the whole space: $\varepsilon(x_1) = (n^{(1)})^2$, if $x_1 > 0$, and $\varepsilon(x_1) = (n^{(2)})^2$, if $x_1 < 0$. Then, in the coordinate system connected with the interface, $\overline{T}_{jk}(\mathbf{x};t)$ will look as follows:

$$\overline{T}_{jk}(\mathbf{x};t) = \begin{pmatrix} \overline{\sigma}_{\alpha\beta}(\mathbf{x};t) & -i\overline{\mathbf{g}}(\mathbf{x};t) \\ -i\overline{\mathbf{s}}(\mathbf{x};t) & \overline{w}(\mathbf{x};t) \end{pmatrix} , \tag{12}$$

where $j,k = 1,2,3,4$, and $\alpha,\beta = 1,2,3$. The constituents of this tensor are as follows. $\overline{w}(\mathbf{x};t)$ is the electromagnetic energy density at an arbitrary point and at an arbitrary instant t:

$$\overline{w}^{(a)}(\mathbf{x}) = \frac{1}{8\pi}\left(\varepsilon(x_1)\left(\overline{\mathbf{E}}^{(a)}(\mathbf{x};t)\right)^2 + \left(\overline{\mathbf{H}}^{(a)}(\mathbf{x};t)\right)^2\right] . \tag{13}$$

$\overline{\mathbf{s}}(\mathbf{x};t)$ is the Poynting vector:

$$\overline{\mathbf{s}}(\mathbf{x};t) = \frac{1}{4\pi}\overline{\mathbf{E}}(\mathbf{x};t) \times \overline{\mathbf{H}}(\mathbf{x};t) . \tag{14}$$

We use the system of units, in which the light velocity in vacuum is unity, in such a system $\overline{\mathbf{s}}^{(a)}(\mathbf{x};t)$ coincides with the density of the linear momentum defined in the sense of Abraham [22, 25, 26]. $\overline{\mathbf{g}}^{(a)}(\mathbf{x};t)$ is the density of the linear momentum defined in the sense of Minkowski [22, 25, 26], in the employed system of units,

$$\overline{\mathbf{g}}(\mathbf{x};t) = \varepsilon(x_1)\overline{\mathbf{s}}(\mathbf{x};t) . \tag{15}$$

$\overline{\sigma}_{\alpha\beta}(\mathbf{x};\mathbf{t})$ is the Maxwellian stress tensor:

$$\overline{\sigma}_{\alpha\beta}(\mathbf{x};t) = \frac{1}{4\pi}\left(\varepsilon(x_1)\overline{E}_\alpha(\mathbf{x};t)\overline{E}_\beta(\mathbf{x};t) + \overline{H}_\alpha(\mathbf{x};t)\overline{H}_\beta(\mathbf{x};t) - \delta_{\alpha\beta}\overline{w}(\mathbf{x};t)\right) , \tag{16}$$

were $\delta_{\alpha\beta}$ is the Kronecker symbol.

By means of the tensor $\overline{T}_{jk}(\mathbf{x};t)$ the local conservation laws of the energy ($j = 4$) and of the components of the Minkowski linear momentum ($j = 1,2,3$) are written as follows:

$$\frac{\partial\overline{T}_{j4}(\mathbf{x};t)}{\partial t} = -i\sum_{\beta=1}^{3}\frac{\partial\overline{T}_{j\beta}(\mathbf{x};t)}{\partial x_\beta} , \tag{17}$$

Eq. (17) is correct in the whole space except the interface and at an arbitrary instant of time. This equation can be used as a basis for the description of the motion of the constructed packet at any t.

Let us first consider quasi-free regimes. In these regimes, only one packet exits in the first medium, i.e., in this medium, $\overline{\mathbf{F}}(\mathbf{x};\tilde{t}) \cong \overline{\mathbf{F}}^{(i)}(\mathbf{x};\tilde{t})$ when $\tilde{t} < 0$, and $\overline{\mathbf{F}}(\mathbf{x};\tilde{t}) \cong \overline{\mathbf{F}}^{(\rho)}(\mathbf{x};\tilde{t})$ when $\tilde{t} > 0$. Let us take this fact into account and integrate Eq. (17) over the upper half-space if $\tilde{t} < 0$ and over the upper and

lower half-spaces separately if $\tilde{t} > 0$. At that let us take into account that $\overline{\mathbf{F}}^{(a)}(\mathbf{x};\tilde{t})$ is negligible on the interface; thus, the integration over the respective half-space may be replaced by the integration over the whole space, and, as a consequence, the partial derivative $\partial/\partial\tilde{t}$ in the left-hand side of the obtained expression may be replaced by the full one $d/d\tilde{t}$. Then we obtain that the electromagnetic energy $\overline{W}^{(a)}(\tilde{t})$ and the Minkowski linear momentum $\overline{\mathbf{G}}^{(a)}(\tilde{t})$ of the a-th packet do not depend on $\tilde{t}$, i.e.

$$\overline{W}^{(a)}(\tilde{t}) = \int \overline{w}^{(a)}(\mathbf{x};\tilde{t})d\mathbf{x} \cong \overline{W}^{(a)} \,, \tag{18}$$

and

$$\overline{\mathbf{G}}^{(a)}(\tilde{t}) = \int \overline{\mathbf{g}}^{(a)}(\mathbf{x};\tilde{t})d\mathbf{x} \cong \overline{\mathbf{G}}^{(a)} \,, \tag{19}$$

where $\overline{W}^{(a)}$ and $\overline{\mathbf{G}}^{(a)}$ are the constants. Again, by using the relation (15) we obtain that the linear momenta of the a-th packet defined in the senses of Abraham and Minkowski are related by

$$\overline{\mathbf{S}}^{(a)}(\tilde{t}) = \int \overline{\mathbf{s}}^{(a)}(\mathbf{x};\tilde{t})d\mathbf{x} \cong \overline{\mathbf{S}}^{(a)} = \frac{\overline{\mathbf{G}}^{(a)}}{(n^{(a)})^2} \,. \tag{20}$$

Let us now consider the 3D radius-vector of the centre of gravity of the a-th packet $\overline{\mathbf{R}}^{(a)}(\tilde{t})$. It is defined as follows:

$$\overline{\mathbf{R}}^{(a)}(\tilde{t}) = \frac{\int \mathbf{x}\overline{w}^{(a)}(\mathbf{x};\tilde{t})d\mathbf{x}}{\overline{W}^{(a)}} \,, \tag{21}$$

the integration in the right-hand side of Eq. (21) being performed over the respective half-space. Let us act by the operator $\partial/\partial\tilde{t}$ on the left-hand and right-hand sides of Eq. (21). In the right-hand side of the obtained expression, let use the relation (17) with $j = 4$, and, by the calculation of the numerator, let us perform the integration by parts. In the left-hand side, the partial derivative $\partial/\partial\tilde{t}$ may be replaced, due to the above reasoning, by the full one $d/d\tilde{t}$. After carrying out these operations, we obtain the following equation of the motion of the a-packet:

$$\frac{d\overline{\mathbf{R}}^{(a)}(\tilde{t})}{d\tilde{t}} = \frac{\overline{\mathbf{S}}^{(a)}}{\overline{W}^{(a)}} \,. \tag{22}$$

Thus, the motion of the a-th packet under condition (9) is rectilinear. Let us impose the following restriction on $\tilde{t}$:

$$\tilde{t} \ll D^2/\lambda \,. \tag{23}$$

If $\tilde{t}$ satisfies the relation (23), the vector $\overline{\mathbf{F}}^{(a)}(\mathbf{x};\tilde{t})$ approximately coincides with the vector $\mathbf{F}^{(a)}(\mathbf{x})$ inside the section of a-th beam, whose centre is situated at

$X_3^{(a)} = \tilde{t}/n^{(a)}$ and whose length equals to $Dn^{(1)}/n^{(a)}$. As a consequence,

$$\overline{\mathbf{R}}_\perp^{(a)}(\tilde{t}) \simeq \mathbf{R}_\perp^{(a)}(\tilde{t}/n^{(a)}) \ , \tag{24}$$

what means that, in the quasi-free regime, the centre of gravity of the i-th packet moves along the axis of the incident beam, while the centre of gravity of s-th packet moves along the s-th actual axis.

Thus, the angular shifts $\phi^{(\rho)}$ and $\phi^{(\tau)}$ are equal to the angles of inclination of the vectors $\overline{\mathbf{S}}^{(\rho)}$ and $\overline{\mathbf{S}}^{(\tau)}$ to the incidence plane. Consider the properties of these vectors. Let us begin with the zero-order approximation with respect to λ/b. In this approximation, the laws of reflection and transmission for the packet under consideration and for the plane wave, which is incident at the angle $\theta^{(i)}$ and whose polarization vector is $\hat{\mathbf{e}}^{(i)} = \overline{\mathbf{E}}^{(i)}(\mathbf{x};t_0)/|\overline{\mathbf{E}}^{(i)}(\mathbf{x};t_0)|$, are equivalent. Let us denote the characteristics of the secondary beams calculated in the zero-order approximation by means of the additional superscript 0. Then we have: $\overline{W}^{(s0)} = Q^{(s)}\overline{W}^{(i)}$, were $Q^{(\rho)}$ and $Q^{(\tau)}$ are the reflectivity and transmissivity of the above-mentioned plane wave [25, 27]. As for the vectors $\overline{\mathbf{S}}^{(\rho 0)}$ and $\overline{\mathbf{S}}^{(\tau 0)}$, they are directed along the geometric optics axes of the reflected and transmitted beams, respectively: $\overline{\mathbf{S}}^{(s0)} = \overline{S}_3^{(s0)}\hat{\mathbf{x}}_3^{(s)}$. The values $\overline{S}_3^{(s0)}$ are related to $\overline{W}^{(s0)}$ by $\overline{S}_3^{(s0)} = \overline{W}^{(s0)}/n^{(s)}$. The $x_1^{(s)}$ and x_2 components of the vector $\overline{\mathbf{S}}^{(s)}$ are of a highest order. In the nearest-order approximation, the former does not affect the angle of inclination of the vector $\overline{\mathbf{S}}^{(s)}$ to the incidence plane. Thus, by using the above relations one obtains:

$$\phi^{(s)} \cong \frac{\overline{S}_2^{(s)}}{S_3^{(s0)}} \cong \frac{n^{(s)}\overline{S}_2^{(s)}}{Q^{(s)}\overline{W}^{(i)}} \ . \tag{25}$$

Eq. (25) shows that from the point of view of the dynamics the calculation of the ATS of the secondary beam $\phi^{(s)}$ turns to the calculation of the transverse linear momentum $\overline{S}_2^{(s)}$, which is obtained by the packet after the reflection or transmission. In order to carry out these calculations, it is necessary to make an assumption about the structure of the incident beam. However, a relation between $\phi^{(\rho)}$ and $\phi^{(\tau)}$ can be obtained without any assumption. The derivation of this relation is carried out on the basis of the conservation law of the transverse component of the linear momentum defined in the sense of Minkowski. In order to do that, let us take $j = 2$ in Eq. (17) and perform the integration of the left-hand and right-hand sides of this equation over the whole space (t is assumed to be arbitrary). By means of the Gauss theorem, the 3D right-hand integral transforms into two 2D integrals of $\overline{\sigma}_{21}(\mathbf{x};t)$ over both sides of the interface. By taking into account the boundary conditions at the interface [25, 27], one can be convinced that $\overline{\sigma}_{21}(\mathbf{x};t)$ is continuous at the interface; as a consequence, the sum of the two above-mentioned surface integrals equals zero, therefore

$d\overline{G}_2(t)/dt = 0$. At the instant t_0, $\overline{G}_2(t_0) \cong \overline{G}_2^{(i)}(t_0) = 0$, hence, $\overline{G}_2(t) = 0$ at an arbitrary t. Let us now consider the secondary packets at a instant $\tilde{t} > 0$, when they are well-formed. At such an instant, $\overline{G}_2(\tilde{t}) = \overline{G}_2^{(\rho)}(\tilde{t}) + \overline{G}_2^{(\tau)}(\tilde{t})$. Hence, taking into account Eq. (19), we get: $\overline{G}_2^{(\rho)} + \overline{G}_2^{(\tau)} = 0$. By using in this expression the relation between the quantities $\overline{G}_2^{(a)}$ and $\overline{S}_2^{(a)}$ given by Eq. (20), one obtains:

$$(n^{(1)})^2 \overline{S}_2^{(\rho)} + (n^{(2)})^2 \overline{S}_2^{(\tau)} = 0 \, . \tag{26}$$

Thus, the reflected and transmitted beams separately can obtain transverse Abraham linear momenta, however, their values must obey the condition (26).

Substitution of the expression for $\overline{S}_2^{(\rho,\tau)}$ obtained from Eq. (25) into Eq. (26) and the using of the relation (7) yields

$$Q^{(\rho)} \Phi^{(\rho)} + Q^{(\tau)} \Phi^{(\tau)} = 0 \, . \tag{27}$$

The relation between the ATSs of the reflected and transmitted beams, $\phi^{(\rho)}$ and $\phi^{(\tau)}$, can be obtained from Eq. (27) by means of Eq. (7). It should be underlined that the relation (27) does not depend on the incident beam's structure. Nevertheless, one can be convinced that the spin-dependent and the spin-independent ATSs, which have been calculated in Refs. [13] and [14, 19, 20], respectively, satisfy the relation (27), although the values of these ATSs depend on the structure mentioned.

The following conclusion can be made exclusively on the basis of the conservation law of the transverse component of the Minkowski linear momentum. The ATS phenomenon can take place only when both the reflected and transmitted beams are generated. Independent of the incident beam's structure, the ATS must be equal zero when one secondary beam is generated, i.e. when $Q^{(\rho)} = 1$, and $Q^{(\tau)} = 0$, or $Q^{(\tau)} = 1$, and $Q^{(\rho)} = 0$; the former situation takes place when $n^{(1)} > n^{(2)}$ and $\theta^{(i)}$ exceeds the critical angle for the total reflection, the latter takes place when the incident beam is p-polarized and $\theta^{(i)}$ is equal to the Brewster angle.

IV. Conclusions

1. We have analyzed the ATSs of the secondary beams on the basis of conservation laws, which take place in the processes of reflection and transmission of a wave packet at a plane interface of two isotropic transparent media. It has been shown that the values of the ATSs of the reflected and transmitted beams, in case the shifts exist, are controlled by the conservation law of the transverse component of the Minkowski linear momentum. Owing to this law, the angles $\Phi^{(\rho)}$ and $\Phi^{(\tau)}$ must satisfy the relation (27).

2. The obtained relation (27) is of a general character: by its derivation, we have assumed that the incident beam is paraxial, but the distribution of the elec-

tromagnetic field inside the beam has not been specified. This fact is remarkable because the ATS of every secondary beam depends on this distribution.

3. Without direct calculations of the ATS, only on the basis of the conservation of the transverse component of the Minkowski linear momentum, one can conclude that the ATS must be equal zero when only one secondary beam is generated, i.e. in the total-reflection and total-transmission regimes.

4. Our analysis was restricted with the processes of reflection and transmission of the light beam at a single interface of two isotropic transparent media. However, the conservation of the transverse component of the Minkowski linear momentum takes also place when the beam is reflected and transmitted at the stratified structures of the isotropic transparent media. If in the course of such a process several well-divided reflected or transmitted beams are generated (see, for instance, [28]), their ATSs must obey a condition similar to the relation (27).

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